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Total No. of Questions : 09]

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B. Tech. (Sem. - 1st)

ENGINEERING MATHEMATICS - I

SUBJECT CODE : AM - 101

Paper ID : [A0111]

[Note : Please fill subject code and paper ID on OMR]

Time : 03 Hours

Maximum Marks : 60

Instruction to Candidates:

1) Section - A is **Compulsory**.
2) Attempt any **Five** questions from Section - B & C.
3) Selecting atleast **Two** questions from Section - B & C.

Section - A

Q1) [Marks : 2 Each]

a) Find the radius of curvature of $y^2 = 4ax$ at the point (x, y).

b) Find the length of arc $y = x^2$ from $x = 0$ to $x = 2$.

c) Find the area of the region enclosed by the curve $y = x^2$ and the lines $x = 0$, $y = 0$ and $x = 2$.

d) If $z = f(x, y)$ is a surface, then what is Geometrical meaning of $\frac{\partial z}{\partial x}$ (partial derivative w.r.t. x).

e) If $u(x, y) = x^y$, find $\frac{\partial^2 u}{\partial x \partial y}$ at (1, 2).

f) Write down the maclaurin's theorem of $e^x \sin y$ up to 2 terms.

g) A plane passes through a fixed point (a, b, c). Show that the locus of the foot of perpendicular to it from the origin is the sphere

$$x^2 + y^2 + z^2 - ax - by - cz = 0.$$

h) Evaluate the integral $\int_1^2 \int_0^x \frac{dxdy}{x^2 + y^2}$.

i) If a series $\sum_{n=1}^{\infty} u_n$ is convergent, then $\lim_{n \to \infty} u_n = 0$.

j) Find the cube roots of unity using De Moivre's theorem.

Section - B

[Marks : 8 Each]

Q2) Trace the curve $x = (y - 1)(y - 2)(y - 3)$.

Q3) Find the length of a loop of the curve $9ay^2 = x(x - 3a)^2$, $a > 0$.

Q4) If $u = \tan^{-1}\left(\frac{y^2}{x}\right)$, show that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\sin 2u \sin^2 u$.

Q5) Find the minimum value of the function $x^2 + y^2 + z^2$ subject to the condition $ax + by + cz = a + b + c$.

Section - C

[Marks : 8 Each]

Q6) A sphere of constant radius 'K' passes through the origin and meets the axes in A, B, C. Prove that the centroid of the triangle lies on the sphere $9(x^2 + y^2 + z^2) = 4K^2$.

Q7) Evaluate $\iint_D e^{-(x^2+y^2)}\,dxdy$, where D is the region bounded by $x^2 + y^2 = a^2$ by changing in polar coordinate.

Q8) Test for convergence the series

$$\frac{\alpha}{\beta} + \frac{1+\alpha}{1+\beta} + \frac{(1+\alpha)(2+\alpha)}{(1+\beta)(2+\beta)} + \text{——}$$

Q9) Show that $2^5 \sin^4\theta \cos^2\theta = \cos 6\theta - 2\cos 4\theta - \cos 2\theta + 2$.

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